Bouncing Ball Problem and Geometric Series
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_A Motivating Example for Module 3_

**Project Description**

This project demonstrates the following concepts in integral calculus:

1. Sequences.
2. Sum of a geometric progression.
3. Infinite series.

**Numeric Example**

In my experiment, the ball was dropped from a height of 6 feet and begins bouncing. The height of each bounce is three-fourths the height of the previous bounce. Find the total vertical distance travelled by the ball.

**Solution** When the ball hits the ground for the first time, it has traveled a distance $D_1 = 6$ feet. For subsequent bounces, let $D_i$ be the distance traveled up and down. For example, $D_2$ and $D_3$ are

$$D_2 = 6 \left( \frac{3}{4} \right) + 6 \left( \frac{3}{4} \right) = 12 \left( \frac{3}{4} \right)$$

up + down

and

$$D_3 = 6 \left( \frac{3}{4} \right) \left( \frac{3}{4} \right) + 6 \left( \frac{3}{4} \right) \left( \frac{3}{4} \right) = 12 \left( \frac{3}{4} \right)^2$$
By continuing this process, it can be determined that the total vertical distance is

\[ D = 6 + 12 \left( \frac{3}{4} \right) + 12 \left( \frac{3}{4} \right)^2 + 12 \left( \frac{3}{4} \right)^3 + ... \]

\[ = 6 + 12 \sum_{n=0}^{\infty} \left( \frac{3}{4} \right)^{n+1} \]

\[ = 6 + 12 \left( \frac{3}{4} \sum_{n=0}^{\infty} \left( \frac{3}{4} \right)^n \right) \]

\[ = 6 + 9 \left[ \frac{1}{1 - (3/4)} \right] \]

\[ = 6 + 9(4) = 42 \text{ feet.} \]

**Your Assignment**

1. Obtain a ball (e.g. a tennis ball or racket ball).
2. Drop the ball from a height \( h \) at your choice.
3. Find the ratio of the maximum height \( h_1 \) to which the ball bounces back to the initial height \( h \) from which the ball was released.
4. Assume that the ratio found in part (3) remains constant for subsequent bounce ups.
5. Obtain a formula that will provide the total vertical distance traveled by the vertically bouncing ball from initial release to a full stop.