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## Calculus in Crash Safety Tests: The Head Injury Criterion(HIC)Number

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# Calculus in Crash Safety Tests: The Head Injury Criterion(HIC)Number

## *A Mini -Project for Module 1*

### Project Description

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This project demonstrates the following concepts in integral calculus:

- 1) The concept of *numerical integration as a discrete and finite Riemann sum* (i.e. the evaluation of a definite integral using a finite set of discrete  $(x, y)$  values rather than the primitive of a function)
- 2) The concept of the *primitive (anti-derivative) of a function* and *specific integration techniques*
- 3) The calculation of the *definite integral using the primitive of a function*

Other key concepts students will be exposed to:

- 1) The practical concept of *functions defined only via their  $(x, y)$  data table* (in this case  $(a, t)$ , rather than via an equation or formula for the function)
- 2) The concept of *a family of functions with a parameter* and their graphs
- 3) The concept of *interpolating a set of experimentally obtained data values* to improve on the precision of the numerical calculations above
- 4) The concept of *data modeling (also known as curve fitting)* in an attempt to come up with an equation (formula) that models the discrete set of data points and generates a continuous curve that best matches them
- 5) The concept of *average value of a function*

Project description.

The quantity known as the Head Injury Criterion (HIC) is in actuality two things in one: **a number** and **a criterion** based on that number.

The **HIC number** is calculated starting from the short-lived but extremely large time-dependent acceleration  $a(t)$  experienced by the head of a person during an impact.

The **HIC criterion** associates different likelihoods of head injury to different ranges of values of the HIC number.

### **What Happens to the Car During a Car Crash?**

- The car itself experiences a very large negative acceleration (tens of times larger than  $g = 9.8 \text{ m/s}^2$ ) due to the almost sudden stop.
- The absolute (positive) value of this negative acceleration is often referred to as the **deceleration** of the car.
- The impact lasts for only a fraction of a second (typical values of **100 to 250 milliseconds** or so). The forces acting on the car during that time are very high and destructive.

### **What Happens to the Driver (and Passengers) During a Car Crash?**

- Various parts of the body will move inside the car and impact the inside of the car at various points.
- The head, in particular, may also experience very large and short-lived accelerations resulting from impacting the steering wheel, the windshield, the side window, head rest.
- These impacts also last for only fractions of a second but can result in serious or deadly head injuries.

### **How is the HIC Number Determined?**

- **In a car safety crash test, inert manufactured bodies (sophisticated mannequins) known as *anthropomorphic test devices*, or simply as *dummies* are placed in the driver's and/or passenger's seat(s).**

<https://www.youtube.com/watch?v=kj9xqrRskrY>

- **To find the HIC number for a particular type of car in a particular type of crash, the head of the dummy is loaded (equipped) with devices called *accelerometers* used to measure and record the large acceleration experienced by the head during that particular crash.**

- The discrete set of acceleration values recorded as a function of time provides the primary data for calculating a HIC number for the car and test in discussion.

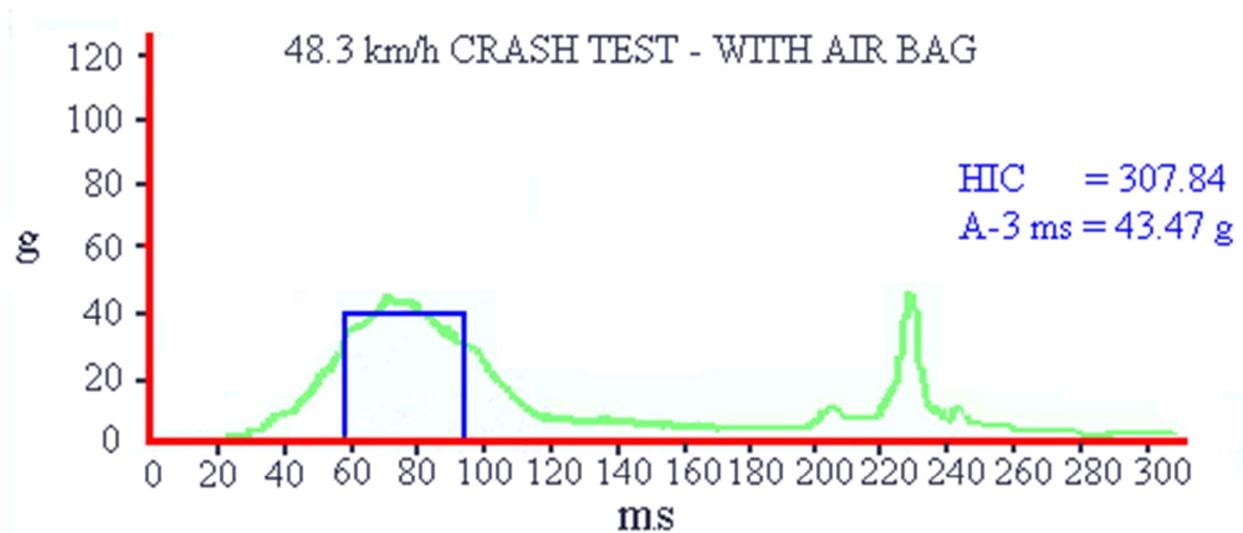
### The Acceleration of the Head and the HIC Number

A representative graph\*: Head acceleration with an air bag

\*From <http://www.intmath.com/applications-integration/hic-head-injury-criterion.php#mercedes>

The vertical axis = the acceleration expressed as multiples of  $g = 9.8 \text{ m/s}^2$

The horizontal axis = the time in milliseconds (the collision begins at  $t = 0$ )



### How is the HIC Number Calculated?

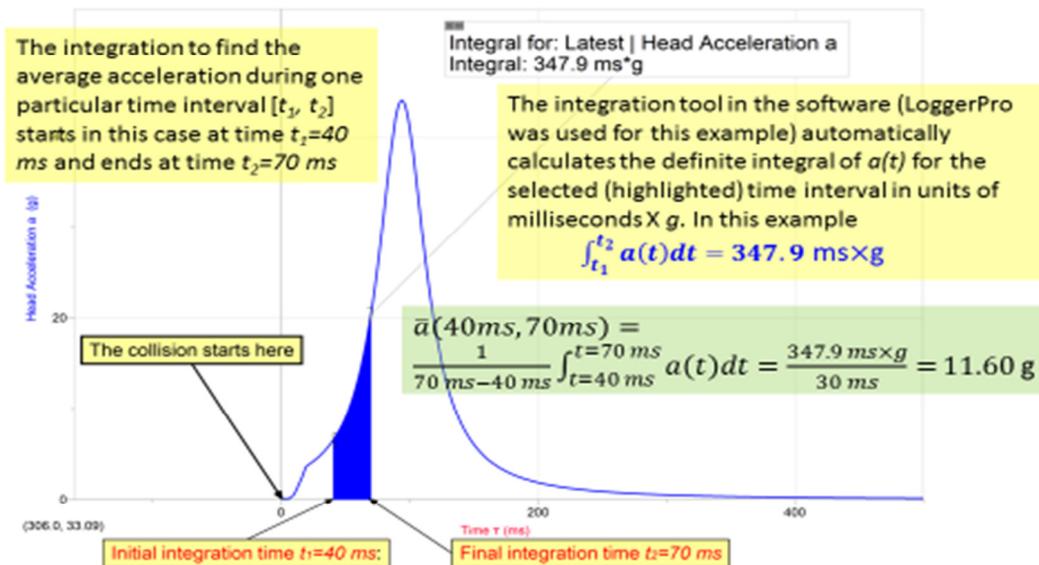
#### STEP 1:

- Starting from the time-dependent acceleration of the head measured during the crash, one calculates several things:
  - The **average acceleration** during *some* time interval  $t_1$  to  $t_2$  of the crash:

$$\bar{a} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} a(t) dt$$

- ❖ Note that  $t_1$  to  $t_2$  are not necessarily the beginning and end instants of the crash. One can (and must) calculate an average acceleration *for any time sub-interval*  $[t_1, t_2]$  during the overall time span of the crash  $[0, T]$ .
- ❖ *Theoretically, that could mean an infinite number of intervals.*
- ❖ In practice, only intervals longer than 3 ms but shorter than 35 ms are used. A 15 ms or a 30 ms time interval seem like good choices.

## Example of a Calculation for Step 1 (one of many calculations needed to find HIC)



Another calculation: Average acceleration for time interval [82ms, 103ms]=41.69g

### How is the HIC Number Calculated?

#### Step 2

- ❑ Once the **average acceleration** for the various time intervals  $t_1$  to  $t_2$  of the crash have been calculated, one calculates the quantities (numbers):

$$(t_2 - t_1) \times (\bar{a})^{2.5} = (t_2 - t_1) \times \left[ \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} a(t) dt \right]^{2.5}$$

- ❖ Again, because an infinite number of time intervals can be theoretically accommodated within the crash interval  $[0, T]$ , this means a large (theoretically infinite) set of such numbers would have to be calculated.

The value of the number calculated above will obviously depend on (and change with) the time interval  $[t_1, t_2]$  chosen.

### Example of calculation for Step 2

- Using the average accelerations calculated earlier for the two time intervals [40 ms, 70 ms] and [82 ms, 103ms], we find

$$\begin{aligned}(t_2 - t_1) \times (\bar{a})^{2.5} &= 30 \text{ ms} \times (11.60 \text{ g})^{2.5} \\ &= 0.03 \text{ s} \times 458.3 \text{ g}^{2.5} = 13.75 \text{ s g}^{2.5}\end{aligned}$$

for the time interval (40 ms, 70 ms) and

$$\begin{aligned}(t_2 - t_1) \times (\bar{a})^{2.5} &= 21 \text{ ms} \times (41.69 \text{ g})^{2.5} \\ &= 0.021 \text{ s} \times 11,222.6 \text{ g}^{2.5} = 235.7 \text{ s g}^{2.5}\end{aligned}$$

for the time interval (82 ms, 103 ms)

- Note that the HIC number is not dimensionless. Rather, it is calculated in **units of seconds  $\times$   $\text{g}^{2.5}$**  where g itself is  $9.8 \text{ m/s}^2$ .

### How is the HIC Number Calculated?

#### Step 3

- Finally, one can find the HIC number:

$$\text{HIC} = \text{Max}[(t_2 - t_1) \times (\bar{a})^{2.5}] = \text{Max}\left\{(t_2 - t_1) \times \left[\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} a(t) dt\right]^{2.5}\right\}$$

- ❖ The maximum, in the above expression, is evaluated with respect to all possible time intervals  $[t_1, t_2]$  during the crash.
- ❖ Simply put, the HIC number is the maximum of all the numbers calculated in step 2 for all possible time intervals  $[t_1, t_2]$  encompassed by the crash interval  $[0, T]$ .
- ❖ In our simple calculation using only two time intervals, the HIC number would be 235.7, but obviously this is only a rough estimate since we have considered only two of the infinite number of time intervals possible.

## Your assignment.

- Download the LoggerPro software by Vernier (our Kettering site license allows the students in PHYS115 and PHYS 225 to download the software free of charge on their own machines. Alternately, Maple or other software may be used (to be investigated; at the instructors choice).
- Go to NHTSA SafeCar.gov Data Base  
<http://www.safercar.gov/Vehicle+Manufacturers>  
and *download a file with the numerical data for the head acceleration vs time* for a specific car type and crash test of your choice. The time has to be in milliseconds and the acceleration in units of  $g$ .
- Use a software package like Maple, LoggerPro or even Excel to create *a graph of discrete points* of acceleration vs time.
- Calculate an approximate value for the area under the entire curve by multiplying each value of the function by the time interval (step) of 1 ms and adding them all up. **(You are calculating a Riemann sum).**
- Calculate the average acceleration during the entire collision time. ( $\bar{a} = \frac{Area}{time}$ )
- Calculate an approximate value for the area under the curve during +/- 15 ms around the peak time by multiplying each value of the function by the time interval (step) of 1 ms and adding them all up.
- Calculate the average acceleration during the entire collision time.
- Using the knowledge gained in steps 3 and 4, perform a systematic calculation of the average acceleration in all 30 ms time intervals using a 10 ms increment for the beginning of the interval starting from  $t = 0$ .
- Create a table of values for the average acceleration vs the beginning time instant. (like the one below)

Initial time t1 (ms)	Final time t2 (ms)	Time Interval (ms)	Average acceleration (g)
0	30	30	2.25
10	40	30	4.08
20	50	30	6.01
30	60	30	8.36

- Use the data in this table to calculate an approximate value for the HIC number for this collision.
- Use the built-in integration function of LoggerPro to compute the value of the area under the entire curve. Compare the results.
- Using the original set of data for  $a$  vs  $t$ , and the software of your choice (LoggerPro, Excel, Maple), perform a curve fitting of the data with a function of the form:

$$a(t) = \frac{b}{(t - c)^2 + d}$$

where  $a$  is the acceleration,  $t$  is the time, and  $b$ ,  $c$  and  $d$  are the fitting parameters whose values will be calculated by the fitting software.

- Note that with a change of variable  $\tau = t - c$  this function is of the form  $a(\tau) = \frac{b}{\tau^2 + d}$  which is further of the form  $f(x) = \frac{1}{x^2 + b^2}$
- Use an appropriate method of integration to find the of your specific best-fit function (i.e. of the function that best models your data for  $a$  vs  $t$ ):

$$\int a(t) dt = \int \frac{b}{(t - c)^2 + d} dt = \text{your answer (a function)}$$

$b$ ,  $c$  and  $d$  are now specific values (numbers) as calculated by the curve fitting software.

- ❑ Calculate the average acceleration in all 30 ms time intervals using a 10 ms increment for the beginning of the interval starting from  $t = 0$  this time using the above definite integral  $\int_t^{t+d} a(t)dt$ .
  
- ❑ Use the above findings to calculate the value for the HIC number for this collision.