Equations and Unknowns

Solve this:

1. There are 6 apples in a basket. Can you divide them between 6 children in such a way so that each child gets one apple and one apple is left in the basket?
2. Two brothers had an argument about whose donkey is slower. They decided to have a race. But when they mounted the donkeys they just sat there because no one wanted to be the first. A mathematician walked by and overheard the argument. She invited the brothers for tea and explained what they needed to do to resolve their issue. After they heard the mathematician speak they jumped on the donkeys and raced as fast as they could. What was the advice of the mathematician?

Equations and their graphical representations.

Problem 1. 79 books are placed on 2 shelves. One shelve has 3 more books than the other. How many books are on each shelve?

Solution using pictures:

_____

_____ + _ _ _

79 – 3 = 76 books

76/2 = 38, 38 books on one shelve and 41 on the other.

Solution using equations:

x = number of books on the shelve with fewer books.

$$\begin{aligned} x + (x + 3) &= 79 \\ x + x + 3 &= 79 \\ 2x + 3 &= 79 \end{aligned}$$

$$\begin{aligned} 2x &= 76 \\ x &= 76/2 \\ x &= 38 \end{aligned}$$

Both solutions find **common** part, x, that is present in the unknown quantities, and the elements that set the quantities apart, here the 3 more books.

We will now solve problems in Section 5 of the book “Mathematical Circle Diaries, Year 1”.

Problem 2. The ages of two siblings add up to 24 years. The brother is 4 years older than the sister. How old are the siblings?

Problem 3. Three consecutive numbers add up to 165, what are these numbers?

Problem 4. A mountain gnome has a collection of precious stones. His dream is to have three times as many stones as he has now. The gnome notices that his dream will come true if he adds 18 stones to his collection. How many precious stones does he have now?

Problem 5. Max and Ann spent the same amount of money at Cedar Point. Max went on 3 coaster rides and one merry-go-round ride, while Ann rode the coaster twice and the merry-go-round 4 times. Which ride was more expensive? Home many times more expensive?

Problem 6. Max and Ann spent the same amount of money at Cedar Point on another day. Max went on 3 coaster rides and two merry-go-round ride, while Ann rode the coaster twice and the merry-go-round 4 times. Which ride was more expensive? Home many times more expensive?

More equations and more unknowns.

Problem 1. Mrs. Daisy has 20 cats, counting the kittens. Every day she gives them a total of 92 cat treats, 5 to every grown cat and 3 to every kitten. How many grown cats and how many kittens does she have?

Since cat gets 3 treats, 20 x 3 60 treats are given away. There remains 92 – 60 = 32 treats given only to adults cats, 2 to each cat, so that there are 32/2 = 16 adult cats and 20 – 16 = 4 kittens.

Solve this as follows:

x = number of adult cats

y = number of kittens

then

$$\begin{aligned} x + y &= 20 \\ 5x + 3y &= 92 \end{aligned}$$

This is a system of two equations with two unknowns. The first solution goes like this:

$$\begin{aligned} 3x + 3y &= 60 \\ 5x + 3y &= 92 \end{aligned}$$

First equation is multiplied by 3. Then first equation is subtracted from the second:

$$2x = 32$$

And solved for x: $x = \frac{32}{2} = 16$ and then for y: $y = 20 - 16 = 4$.

Three elementary row operations can be performed:

1. Equations can be switched, the first can become the second, the second can become the first.
2. An equation can be multiplied by a non-zero constant.
3. A non-zero multiple of an equation can be added to or subtracted from another equation.

Let's see what happened:

$\begin{bmatrix} 1 & 1 & 20 \\ 5 & 3 & 92 \end{bmatrix} R1\ x\ 3 \rightarrow \begin{bmatrix} 3 & 3 & 60 \\ 5 & 3 & 92 \end{bmatrix} R2 - R1 \begin{bmatrix} 3 & 3 & 60 \\ 2 & 0 & 32 \end{bmatrix} R2/2 \begin{bmatrix} 3 & 3 & 60 \\ 1 & 0 & 16 \end{bmatrix} R1/3 \begin{bmatrix} 1 & 1 & 20 \\ 1 & 0 & 16 \end{bmatrix} R1 - R2 \begin{bmatrix} 0 & 1 & 4 \\ 1 & 0 & 16 \end{bmatrix} switch \begin{bmatrix} 1 & 0 & 16 \\ 0 & 1 & 14 \end{bmatrix}$

Observe $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 16 \\ 4 \end{bmatrix}$ or x = 16 and y = 4.

Let's do it again in a more organized way

$$\begin{bmatrix}1 & 1 & 20\\ 5 & 3 & 92\end{bmatrix} eliminate\ the\ 5{:}\ R2 \to R2 - 5R1 \begin{bmatrix}1 & 1 & 20\\ 0 & -2 & -8\end{bmatrix} make\ the\ -2\ into\ 1{:}\ R2$$
$$\to \frac{R2}{(-2)}\begin{bmatrix}1 & 1 & 20\\ 0 & 1 & 4\end{bmatrix} eliminate\ the\ one\ in\ the\ second\ column\ of\ R1{:}\ R1$$
$$\to R1 - R2\begin{bmatrix}1 & 0 & 16\\ 0 & 1 & 4\end{bmatrix}$$

Problem 2. Old MacDonald has 32 animals, all pigs and hens. Together, these animals have 138 legs. How many pigs and how many hens does he have?

x = number of pigs; y = number of hens

Problem 3. A cookie costs twice as much as a piece of candy. Tim bought six cookies and three pieces of candy. Allie bought three cookies and six pieces of candy. Tim paid $1.80 more than Allie. What are the prices of cookies and candy?

Problem 4. A florist is making 5 identical bridesmaid bouquets for a wedding. She has $610 to spend (including tax) and wants 24 flowers for each bouquet. Roses cost $6 each, tulips cost $4 each, and lilies cost $3 each. She wants to have twice as many roses as the other 2 flowers combined in each bouquet. How many roses, tulips, and lilies are in each bouquet?

Problem 5. Rooney's company decides to buy some keyboards, mouse cursors and PC Cameras. They have a budget of $1500 to spend on $30 keyboards, $20 mice, and $50 cameras. Additionally, the number of cursors should be equal to that of keyboards and twice the number of cameras.

How many of each item should he buy?