Problem Set 9

Problem 9.1. Which is larger, $1+2+4+8+16+32+64+128+256+512+1024$ or 2048? By how much?

Problem 9.2. Each day at noon, a steamship leaves Savannah for Belfast while a steamship of the same line leaves Belfast for Savannah. Each ship spends exactly seven 24-hour days at sea, and all travel along the same route. How many Savannah to Belfast ships will a Belfast to Savannah ship meet while underway?

Problem 9.3. The length of one side of a triangle is 3.8 inches, and the length of another side is 0.6 inches. If the third side is a whole number of inches, find its length.

Problem 9.4. Simplify the fraction

$$\frac{1\cdot 2\cdot 3+2\cdot 4\cdot 6+4\cdot 8\cdot 12+7\cdot 14\cdot 21}{1\cdot 3\cdot 5+2\cdot 6\cdot 10+4\cdot 12\cdot 20+7\cdot 21\cdot 35}.$$

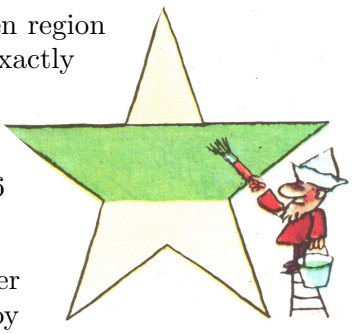
Problem 9.5. Using a pencil, an unmarked ruler, and a sheet of graph paper, how can you draw a square with area (a) double; (b) 5 times larger than the area of one square of the grid?

Problem 9.6. Which is greater, the sum of the lengths of the sides of a quadrilateral, or the sum of the lengths of its diagonals?

Problem 9.7. The menu in a school cafeteria always has the same 10 different items. To vary his meals, George decides to buy a different selection for every lunch. He can eat anywhere from 0 to 10 different items for lunch. (a) For how many days can he eat without repeating a selection? (b) What is the total number of items he will eat in that time?

Problem 9.8. Is it possible to write more than 50 different two-digit numbers on a blackboard without having two numbers on the board whose sum is 100?

Additional Problems



Problem 9.9. Show that the area of the green region of the regular pentagonal star in the picture is exactly half of the total area.

Problem 9.10. Find the sum

$$6 + 66 + 666 + 6666 + 66666 + \cdots + 66\ldots66$$

if the last string of 6's has 100 digits.

Problem 9.11. Is it possible to find a number of the form $11\ldots1100\ldots00$ that is divisible by 2003?

Problem 9.12. A straight bar of length 2 m is cut into five pieces with each piece at least 17 cm long. Prove that there are three of these pieces that can be put together to form a triangle.