

Kettering University Mathematics Olympiad For High School Students 2002

1. The expression $3 + 2\sqrt{2}$ can be represented as a perfect square: $(3 + 2\sqrt{2}) = (1 + \sqrt{2})^2$.
 - (a) Represent $29 - 12\sqrt{5}$ as a perfect square.
 - (b) Represent $10 - 6\sqrt{3}$ as a perfect cube.
2. Find all values of the parameter c for which the following system of equations has no solutions.

$$\begin{cases} x+cy = 1 \\ cx+9y = 3 \end{cases}$$

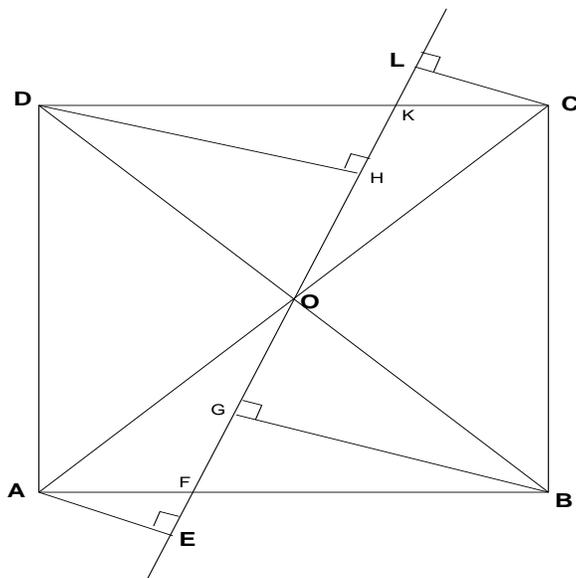
3. The equation $y = x^2 + 2ax + a$ represents a parabola for all real values of a .
 - (a) Prove that each of these parabolas pass through a common point and determine the coordinates of this point.
 - (b) The vertices of the parabolas lie on a curve. Prove that this curve is a parabola and find its equation.
4. Miranda is a 10th grade student who is very good in mathematics. In fact she just completed an advanced algebra class and received a grade of A^+ . Miranda has five sisters, Cathy, Stella, Eva, Lucinda, and Dorothea. Miranda made up a problem involving the ages of the six girls and dared Cathy to solve it.

Miranda said: "The sum of our ages is five times my age. (By 'age' throughout this problem is meant 'age in years'.) When Stella is three times my present age, the sum of my age and Dorothea's will be equal to the sum of the present ages of the five of us; Eva's age will be three times her present age; and Lucinda's age will be twice Stella's present age, plus one year. How old are Stella and Miranda?"

"Well, Miranda, could you tell me something else?"

"Sure", said Miranda, "my age is an odd number".

5. Cities **A**, **B**, **C** and **D** are located in vertices of a square with the area 10,000 square miles. There is a straight-line highway passing through the center of a square. Find the sum of squares of the distances from the cities of to the highway.



6. (a) Among three similar coins there is one counterfeit. It is not known whether the counterfeit coin is lighter or heavier than a genuine one (all genuine coins weight the same). Using two weightings on a pan balance, how can the counterfeit be identified and in process determined to be lighter or heavier than a genuine coin?
- (b) There is one counterfeit coin among 12 similar coins. It is not known whether the counterfeit coin is lighter or heavier than a genuine one. Using three weightings on a pan balance, how can the counterfeit be identified and in process determined to be lighter or heavier than a genuine coin?