

Kettering University Mathematics Olympiad For High School Students 2003

1. How many real solutions does the following system of equations have? Justify your answer.

$$\begin{cases} x + y = 3 \\ 3xy - z^2 = 9. \end{cases}$$

2. After the first year the bank account of Mr. Money decreased by 25%, during the second year it increased by 20%, during the third year it decreased by 10%, and during the fourth year it increased by 20%. Does the account of Mr. Money increase or decrease during these four years and how much?
3. Two circles are internally tangent. A line passing through the center of the larger circle intersects it at the points A and D . The same line intersects the smaller circle at the points B and C . Given that $|AB| : |BC| : |CD| = 3 : 7 : 2$, find the ratio of the radiuses of the circles.
4. Find all integer solutions of the equation:

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{19}.$$

5. Is it possible to arrange the numbers $1, 2, \dots, 12$ along the circle so that the absolute value of the difference between any two numbers standing next to each other would be either 3, or 4, or 5? Prove your answer.
6. Nine rectangles of the area 1 sq. mile are located inside the large rectangle of the area 5 sq. miles. Prove that at least two of the rectangles (internal rectangles of area 1 sq. mile) overlap with an overlapping area greater than or equal to $\frac{1}{9}$ sq. mile.