Kettering University Mathematics Olympiad For High School Students 2008

1. The case of Mr. Brown, Mr. Potter, and Mr. Smith is investigated. One of them has committed a crime. Everyone of them made two statements.

   Mr. Brown: I have not done it. Mr. Potter has not done it.

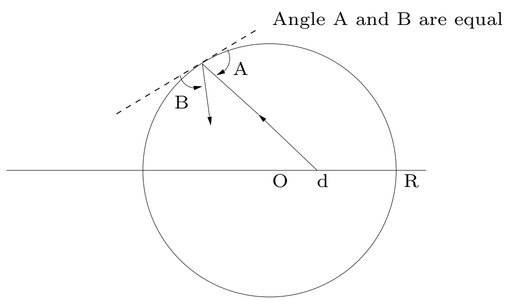
   Mr. Potter: Mr. Brown has not done it. Mr. Smith has done it.

   Mr. Smith: I have not done it. Mr. Brown has done it.

   It is known that one of them told the truth both times, one lied both times, and one told the truth one time and lied one time. Who has committed the crime?

2. Is it possible to draw in a plane 1000001 circles of the radius 1 such that every circle touches exactly three other circles?

3. Consider a circle of radius R centered at the origin. A particle is "launched" from the x-axis at a distance, d, from the origin with $0 < d < R$, and at an angle, α, with the x-axis. The particle is reflected from the boundary of the circle so that **the angle of incidence** equals **the angle of reflection**. Determine the angle α so that the path of the particle contacts the circle's interior at exactly eight points. Please note that α should be determined in terms of the qunatities R and d.



4. Is it possible to find four different real numbers such that the cube of every number equals the square of the sum of the three others?

5. The Fibonacci sequence $(1, 2, 3, 5, 8, 13, 21, \ldots)$ is defined by the following formula: $f_n = f_{n-2} + f_{n-1}$, where $f_1 = 1$, $f_2 = 2$. Prove that any positive integer can be represented as a sum of **different** members of the Fibonacci sequence.

6. 10,000 points are arbitrary chosen inside a square of area 1 m^2. Does there exist a broken line connecting all these points, the length of which is less than 201 m?