Kettering University Mathematics Olympiad For High School Students 2009

1. Prove that if $a, b, c, d$ are real numbers, then
   \[ \max\{a + c, b + d\} \leq \max\{a, b\} + \max\{c, d\} \]

2. Find the smallest positive integer whose digits are all ones which is divisible by 3333333.

3. Find all integer solutions of the equation
   \[ \sqrt{x} + \sqrt{y} = \sqrt{2560}. \]

4. Find the irrational number:
   \[ A = \sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{2} + \cdots}}} \]

   (n square roots).

5. The Math country has the shape of a regular polygon with $N$ vertexes. $N$ airports are located on the vertexes of that polygon, one airport on each vertex. The Math Airlines company decided to build $K$ additional new airports inside the polygon. However the company has the following policies: (i) it does not allow three airports to lie on a straight line, (ii) any new airport with any two old airports should form an isosceles triangle. How many airports can be added to the original $N$?

6. The area of the union of the $n$ circles is greater than $9m^2$ (some circles may have non-empty intersections). Is it possible to choose from these $n$ circles some number of non-intersecting circles with total area greater than $1m^2$?