The Empire Strikes Again

Problem 1. An evil galactic empire is attacking the planet Naboo with numerous automatic drones. The fleet defending the planet consists of 101 ships. By the decision of the commander of the fleet, some of these ships will be used as destroyers equipped with one rocket each or as rocket carriers that will supply destroyers with rockets. Destroyers can shoot rockets so that every rocket destroys one drone. During the attack each carrier will have enough time to provide each destroyer with one rocket but not more. How many destroyers and how many carriers should the commander assign to destroy the maximal number of drones and what is the maximal number of drones that the fleet can destroy?

Problem 2. Solve the inequality: $\sqrt{x^2 - 3x + 2} \leq \sqrt{x + 7}$.

Problem 3. Find all positive real numbers x and y that satisfy the following system of equations:
$$\begin{cases} x^y = y^{x-y} \\ x^x = y^{12y} \end{cases}$$

Problem 4. A convex quadrilateral $ABCD$ with sides $AB = 2$, $BC = 8$, $CD = 6$, and $DA = 7$ is divided by a diagonal AC into two triangles. A circle is inscribed in each of the obtained two triangles. These circles touch the diagonal at points E and F. Find the distance between the points E and F.

Problem 5. Find all positive integer solutions n and k of the following equation:
$$\underbrace{11\ldots1}_{n}\underbrace{00\ldots0}_{2n+3} + \underbrace{77\ldots7}_{n+1}\underbrace{00\ldots0}_{n+1} + \underbrace{11\ldots1}_{n+2} = 3k^3.$$

Problem 6. The Royal Council of the planet Naboo consists of 12 members. Some of these members mutually dislike each other. However, each member of the Council dislikes less than half of the members. The Council holds meetings around the round table. Queen Amidala knows about the relationship between the members so she tries to arrange their seats so that the members that dislike each other are not seated next to each other. But she does not know whether it is possible. Can you help the Queen in arranging the seats? Justify your answer.