

Bilbo's New Adventures

Problem 1. Solve the equation: $\sqrt{x} + \sqrt{x+1} - \sqrt{x+2} = 0$.

Problem 2. Solve the inequality: $\ln(x^2 + 3x + 2) \leq 0$.

Problem 3. In the trapezoid $ABCD$ ($AD \parallel BC$) $|AD| + |AB| = |BC| + |CD|$. Find the ratio of the length of the sides AB and CD ($|AB|/|CD|$).

Problem 4. Gollum gave Bilbo a new riddle. He put 64 stones that are either white or black on an 8×8 chess board (one piece per each of 64 squares). At every move Bilbo can replace all stones of any horizontal or vertical row by stones of the opposite color (white by black and black by white). Bilbo can make as many moves as he needs. Bilbo needs to get a position when in every horizontal and in every vertical row the number of white stones is greater than or equal to the number of black stones. Can Bilbo solve the riddle and what should be his solution?

Problem 5. Two trolls Tom and Bert caught Bilbo and offered him a game. Each player got a bag with white, yellow, and black stones. The game started with Tom putting some number of stones from his bag on the table, then Bert added some number of stones from his bag, and then Bilbo added some stones from his bag. After that three players started making moves. At each move a player chooses two stones of different colors, takes them away from the table, and puts on the table a stone of the color different from the colors of chosen stones. Game ends when stones of one color only remain on the table. If the remaining stones are white Tom wins and eats Bilbo, if they are yellow, Bert wins and eats Bilbo, if they are black, Bilbo wins and is set free. Can you help Bilbo to save his life by offering him a winning strategy?

Problem 6. There are four roads in Mirkwood that are straight lines. Bilbo, Gandalf, Legolas, and Thorin were travelling along these roads, each along a different road, at a different constant speed. During their trips Bilbo met Gandalf, and both Bilbo and Gandalf met Legolas and Thorin, but neither three of them met at the same time. When meeting they did not stop and did not change the road, the speed, and the direction. Did Legolas meet Thorin? Justify your answer.