Logic, Mathematical Logic, or Common Sense?

Problem 1. A dealer bought an article for $7, sold it for $8, bought it back for $9, and sold it for $10. How much profit did he make?

Problem 2. A bottle of wine cost ten dollars. The wine was worth nine dollars more than the bottle. How much was the bottle worth?

Problem 3. Three friends- Michael, Peter, and John - were all perfect logicians. Each could instantly deduce all consequences of any set of premises. Also, each was aware that each of the others was a perfect logician. The three were shown seven stamps: two red ones, two yellow ones, and three green ones. They were then blindfolded, and a stamp was pasted on each of their foreheads; the remaining four stamps were placed in a drawer. When the blindfolds were removed, Michael was asked, "Do you know one color that you definitely do not have?" Michael replied, "No." Then Peter was asked the same question and replied, "No." Is it possible, from this information, to deduce the color of Michael's stamp, or of Peter's, or of John's?

Problem 4. The king pointed to the signs on the doors of the rooms:

| I
IN THIS ROOM THERE IS A LADY, AND IN THE OTHER ROOM THERE IS A TIGER | | II
IN ONE OF THESE ROOMS THERE IS A LADY, AND IN ONE OF THESE ROOMS THERE IS A TIGER |

"Is it true, what the signs say?" asked the prisoner.
"One of them is true," replied the king, "but the other one is false."
If you were the prisoner, which door would you open (assuming, of course, that you preferred the lady to the tiger)?   It is possible that both rooms contain ladies or both rooms contain tigers, or that one room contains a lady and the other a tiger.

In the next two problems either both signs are true or both are false.

Problem 5.

| I
AT LEAST ONE OF THESE ROOMS CONTAINS A LADY | | II
A TIGER IS IN THE OTHER ROOM |

Problem 6.

| I
EITHER A TIGER IS IN THIS ROOM OR A LADY IS IN THE OTHER ROOM | | II
A LADY IS IN THE OTHER ROOM |

Well, in each of the trials of this day, the king explained that in the left-hand room (Room I), if a lady is in it, then the sign on the door is true, but if a tiger is in it, the sign is false. In the right-hand room (Room II), the situation is the opposite: a lady in the room means the sign on the door is false, and a tiger in the room means the sign is true. Again, it is possible that both rooms contain ladies or both rooms contain tigers, or that one room contains a lady and the other a tiger.

Problem 7. After the king explained the above rules to the prisoner, he pointed to the two signs:

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┌─────────────────────┐        ┌─────────────────────┐
│          I          │        │          II         │
│                     │        │                     │
│     BOTH ROOMS      │        │     BOTH ROOMS      │
│   CONTAIN LADIES    │        │   CONTAIN LADIES    │
└─────────────────────┘        └─────────────────────┘
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Problem 8.

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┌─────────────────────┐        ┌─────────────────────┐
│          I          │        │          II         │
│                     │        │                     │
│  AT LEAST ONE ROOM  │        │   THE OTHER ROOM    │
│   CONTAINS A LADY   │        │   CONTAINS A LADY   │
└─────────────────────┘        └─────────────────────┘
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Problem 9.

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┌─────────────────────┐        ┌─────────────────────┐
│          I          │        │          II         │
│     IT MAKES NO     │        │                     │
│     DIFFERENCE      │        │   THERE IS A LADY   │
│   WHICH ROOM YOU    │        │   IN THE OTHER ROOM │
│        PICK         │        │                     │
└─────────────────────┘        └─────────────────────┘
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Problem 10.

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┌─────────────────────┐        ┌─────────────────────┐
│          I          │        │          II         │
│   IT DOES MAKE A    │        │                     │
│  DIFFERENCE WHICH   │        │    YOU ARE BETTER   │
│   ROOM YOU PICK     │        │   OFF CHOOSING THE  │
│                     │        │     OTHER ROOM      │
└─────────────────────┘        └─────────────────────┘
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Problem 11. "There are no signs above the doors! " exclaimed the prisoner.

"Quite true," said the king. "The signs were just made, and I haven't had time to put them up yet. Only remember, of course, a lady in the left hand room means the sign which should be on that door is true and a tiger in it means the sign should be false, and that the reverse is true for the right-hand room."

THIS ROOM CONTAINS A TIGER

BOTH ROOMS CONTAIN TIGERS

Boolean Functions and Formulas

Conjunction: A & B (A and B)

A \ B	1	0
1	1	0
0	0	0

Disjunction   A ^ B (A or B)

A \ B	1	0
1	1	1
0	1	0

Implication A → B

A \ B	1	0
1	1	0
0	1	1

Equivalence   A ↔ B

A \ B	1	0
1	1	0
0	0	1

Negation $\bar{A}$

A	1	0
$\bar{A}$	0	1

Problem 12-19. Solve problems 4-11 using the Boolean functions.

Problem 20. A traveler visited an island where two tribes lived. The people of the first tribe always said the truth, and the people of the second tribe always lied. The traveler had to choose one of the two roads and he did not know which road is the road to the city (he knew that only one of these two roads is the correct one). The traveler saw a man walking nearby and he decided to ask this man, but he did not know the tribe this man belonged. Nevertheless he asked only one question such that the answer gave him the right direction. What was the question?

References.

1. The lady, or the tiger? by Raymond Smullyan, Times Books, N.Y.,1992.