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Bouncing Ball Problem and Geometric Series

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Bouncing Ball Problem and Geometric Series

A Motivating Example for Module 3

Project Description

This project demonstrates the following concepts in integral calculus:

1. Sequences.
2. Sum of a geometric progression.
3. Infinite series.

Numeric Example

In my experiment, the ball was dropped from a height of 6 feet and begins bouncing. The height of each bounce is three-fourths the height of the previous bounce. Find the total vertical distance travelled by the ball.

Solution When the ball hits the ground for the first time, it has traveled a distance $D_1 = 6$ feet. For subsequent bounces, let D_i be the distance traveled up and down. For example, D_2 and D_3 are

$$D_2 = 6\left(\frac{3}{4}\right) + 6\left(\frac{3}{4}\right) = 12\left(\frac{3}{4}\right)$$

up + down

and

$$D_3 = 6\left(\frac{3}{4}\right)\left(\frac{3}{4}\right) + 6\left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = 12\left(\frac{3}{4}\right)^2$$

By continuing this process, it can be determined that the total vertical distance is

$$D = 6 + 12\left(\frac{3}{4}\right) + 12\left(\frac{3}{4}\right)^2 + 12\left(\frac{3}{4}\right)^3 + \ldots$$

$$= 6 + 12\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^{n+1}$$

$$= 6 + 12\left(\frac{3}{4}\right)\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^{n}$$

$$= 6 + 9\left[\frac{1}{1-(3/4)}\right]$$

$= 6 + 9(4) = 42$ feet.

Your Assignment

1. Obtain a ball (e.g. a tennis ball or racket ball).
2. Drop the ball from a height h at your choice.
3. Find the ratio of the maximum height h_1 to which the ball bounces back to the initial height h from which the ball was released.
4. Assume that the ratio found in part (3) remains constant for subsequent bounce ups.
5. Obtain a formula that will provide the total vertical distance traveled by the vertically bouncing ball from initial release to a full stop