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Application to Radioactive Decay-Cancer Treatment

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Application to Radioactive Decay-Cancer Treatment

A Motivating Example for Module 2

Project Description

This project demonstrates the following concepts in integral calculus:

1. Definite Integrals
2. Improper Integrals

Project description.

Cancer Treatment. Implants of iodine-125 with a half - life of 60.1 days are used to treat prostate cancer. The implants are left in the patient and never removed. The amount of energy from the implant that is transmitted to the body is measured in rem units and is given by $\int_0^a P_0 e^{-kt}\,dt$, where k is the decay constant for the radioactive material, a is the number of years since the implant, and P_o is the initial rate that energy is being transmitted

(a) Find the decay rate k of iodine – 125.

(b) How much energy (measured in rems) is given off the first month if the initial energy rate is 10 rem per year?

(c) What is the total amount of energy that the implant will give off over all time?

Numerical Example:

Radioactive Decay Energy. Plutonium has a decay rate of 0.00286% per year. The amount of energy released from a radioactive sample is measured in rem units and is given by $\int_0^a P_0 e^{-kt}\,dt$, where k is the decay rate, a is the number of years, and P_0 is the initial rate of released energy.
For a given sample of plutonium, P_0 = 150 rem/year

(a) Find the number of rems released the first 100 years.
(b) Compute the total number of rems released from the present time on.

Solution

(a) $\int_0^{100} P_0 e^{-kt}\,dt = \int_0^{100} 150 e^{-0.0000286t}\,dt$

$$= 150 \int_0^{100} e^{-0.0000286t}\,dt$$

$$= -\frac{150}{0.0000286} e^{-0.0000286t} \Big|_0^{100}$$

$$= -\frac{150}{0.0000286}\left(e^{-0.003} - e^{0}\right)$$

$$\approx 14979$$

(b) To compute the energy released from the present time on, we integrate from 0 to infinity.

$$\int_0^{\infty} P_0 e^{-kt}\,dt = \lim_{b \to \infty} \int_0^{b} P_0 e^{-kt}\,dt$$

$$= \lim_{b \to \infty} \int_0^{b} 150 e^{-0.0000286t}\,dt$$

$$= \lim_{b \to \infty} -\frac{150}{0.0000286} e^{-0.0000286t} \Big|_0^b$$

$$= -5{,}240{,}000 \lim_{b \to \infty} \left(e^{-0.0000286b} - e^{0}\right)$$

$$= 5{,}240{,}000.$$

Approximately, 5,240,000 rem will be released from this sample.