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# Applications of Improper Integrals to Business and Economics - Capital Value



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# Applications of Improper Integrals to Business and Economics - Capital Value

## *A Mini Project for Module 2*

### Project Description

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This project demonstrates the following concepts in integral calculus:

1. Definite Integrals
2. Improper Integrals

Project description.

The definite integral can be used in a number of applications in business and economics. For example, the definite integral can be used to find the total income over a fixed number of years from a **continuous income stream**.

The definite integral can also be used to find the present value - the value today of a continuous income stream that will be providing income in the future. The present value is useful in deciding when to replace machinery or what new equipment to select.

When the notion of the present value is extended to an infinite time interval, the result is called the capital value of the income stream and is given by

$$\text{Capital value} = \int_0^{\infty} f(t)e^{-rt} dt$$

where  $f(t)$  is the annual rate of flow at time  $t$ , and  $r$  is the annual interest rate, compounded continuously .

### Numerical Example

Suppose that an organization wants to establish a trust fund that will provide a continuous income stream with an annual rate of flow at time  $t$  given by  $f(t) = 10,000$ . If the interest rate remains at 10%, compounded continuously, find the capital value of the fund.

### Solution

The capital value of the fund is given by

$$\begin{aligned}\int_0^{\infty} f(t)e^{-rt} dt &= \int_0^{\infty} 10,000 e^{-rt} dt = \lim_{t \rightarrow \infty} \int_0^t 10,000 e^{-0.10t} dt \\ &= \lim_{t \rightarrow \infty} \left[ -100,000 e^{-0.10t} \right]_0^t \\ &= \lim_{t \rightarrow \infty} \left( \frac{-100,000}{e^{0.10t}} + 100,000 \right) \\ &= 100,000\end{aligned}$$

Thus the capital value of the fund is \$100,000

## Your assignment

The student must find a business that can provide its continuous income stream and the annual rate of flow, or learn of donors that, let's say, provide a cash gift to a hospital or to other charities that generates a continuous income stream. Once the necessary information is found, find the capital value of the business/perpetuity.