6/9/2016

Designing a Roller Coaster

Yaomin Dong

-

Designing a Roller Coaster

A Mini Project for Module 1

Project Description

This project demonstrates the following concepts in integral calculus:

1. Graphical calculus.
2. Modeling.
3. Approximation and estimation

Project description

This project uses the concepts of increasing, decreasing, and concavity to analyze the path of a roller coaster. Students construct graphs of the first and second derivatives of the path of the roller coaster and can use these graphs to find the steepest part of the path. They also relate the slope of the path to the angle the path makes with the horizontal line. The project has the following objectives:

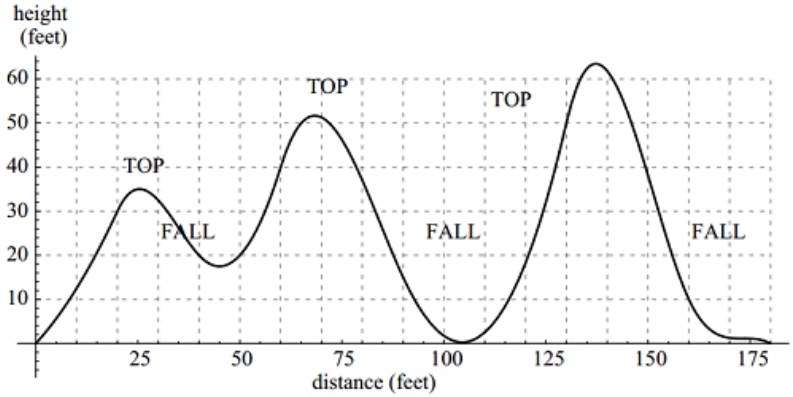
1. Working with graphs.
2. Relating slope with angle.
3. Constructing graphs of first and second derivatives from a given graph.
4. Modeling and optimizing.
5. Using slope graph to find max and min (discovery approach).

Notes

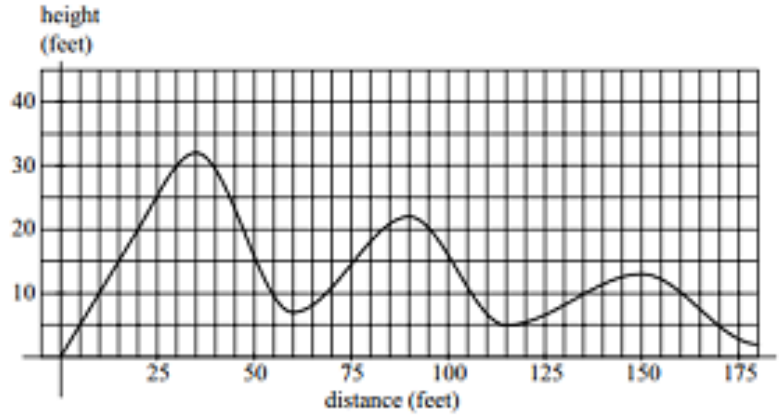
1. This project gives students an outline to follow for much of the problem, but also includes some open-ended pieces.
2. The definition of THRILL is arbitrary. You can replace it if you wish. The given definition implies that a coaster with several small hills (a “kiddie coaster”) is more thrilling than that with a few long steep drops.

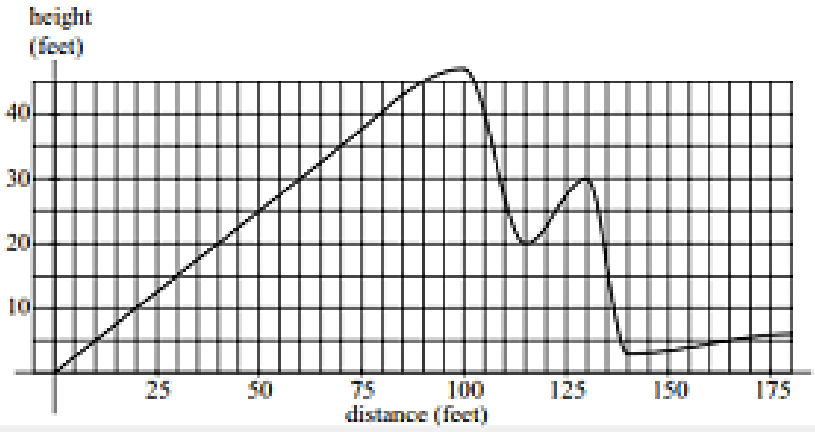
Part 1

Each individual has been given a path design for a straight stretch (no turns) of a proposed roller coaster. There is a support every 10 feet. A safety rule is that a descent can be no steeper than 80^{o} at any point. In addition each design starts with a 45^{o} incline. (Angles refer to the angle that the path makes with a horizontal line.)



Sample roller coaster





Each student will report on her/his design. Your report should include the following data:

1. Where is the path increasing and decreasing? Give your answer in terms of distance along the ground from the start.
2. For each fall, where is the steepest descent and how steep is the angle at that point?
3. Does your path satisfy the safety criterion? Explain why yes or why not.
4. Draw the graph of the slope of the path versus the distance along the ground from the start.
5. The thrill of the coaster is defined as the sum of the angles of the steepest descents in each fall in radians + number of tops. Calculate the thrill of your path.
6. The amount of materials needed for a support is the square of the height of the support. Find the amount of material needed for the supports in your path.

Part 2

Your group should write a report deciding which of the individual paths:

1. Is the most thrilling.
2. Uses the least material for supports.

Part 3

1. Compute how far the coaster would travel along each of the paths your group received in Part 1.
2. If your coaster must start and finish on the ground and be at least 20 feet high at some point, design the coaster that requires the least amount of support materials.
3. Design a path that your group thinks would be the “best” roller coaster if you have 50,000 feet of support material available. Be sure to explain why you think it is the best.

Part 4 (Bonus)

How fast will the coaster travel?