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Period of a Pendulum

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Period of a Pendulum

A Mini Project for Module 3

Project Description

This project demonstrates the following concepts in integral calculus:

1. Techniques of integration.
2. Infinite series.
3. Maclaurin series.
4. Binomial series.

Project description.

The period of a pendulum with length L and the maximum angle $\theta_0$with the vertical is

$$T = 4\sqrt{\frac{L}{g}}\int_0^{\pi/2} \frac{dx}{\sqrt{1 - k^2 sin^2 x}}$$

where $k = sin\left(\theta_0/2\right)$ and g is the acceleration due to gravity.

1. Show that for even powers if sine,

$$\int_0^{\pi/2} sin^{2n}(x)dx = \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{2 \cdot 4 \cdot 6 \cdot 2n}\frac{\pi}{2}$$

2. Expand the integrand as a binomial series and use the result in (1) to show that

$$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1^2}{2^2}k^2 + \frac{1^2 3^2}{2^2 4^2}k^4 + \frac{1^2 3^2 5^2}{2^2 4^2 6^2}k^6 + \cdots\right)$$

3. Explain why for small values of θ_0 the following approximation is frequently used,

$$T \approx 2\pi\sqrt{\frac{L}{g}}$$

4. Obtain real values of L and θ_0. How does the true value of T compares to its approximation?
5. Use of $L = 1$ meter and $\theta_0 = 40^{\circ}$. How does the true value of T compares to its approximation?