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Application of Calculus to Probability



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A Mini Project for Module 2

Project Description

This project demonstrates the following concepts in integral calculus:

1. Definite Integrals.
2. Improper Integrals.

Project description.

The probability, $P(a \leq X \leq b)$, is a measure of the likelihood that an outcome of an experiment will lie between a and b . This probability is given by a definite integral

$$P(a \leq X \leq b) = \int_a^b f(x) dx .$$

The function $f(x)$ that determines the above probability for each a and b is called the *probability density function* of X (or of the experiment whose outcome is X). It is very important to understand that the area under the curve of a probability density function equals (must be) 1. This condition is what relates a density function to the outcome X of a specific experiment.

Experiment has shown that the lifetime of a light bulb is exponentially distributed with probability density function

$$f(x) = \lambda e^{-\lambda x}$$

where λ is a positive constant that may be interpreted as

$\lambda = 1/a$, where $a =$ average value of X .

Example:

Let X be the lifetime of a light bulb of my choice. For simplicity, you may want to measure the lifetime in years, rather than hours. For the type of lightbulb I picked, I find out that the average lightbulb burns out in $1/4$ year of continuous use.

- (a) What proportion of the lightbulbs will burn out within $1/2$ year.
(b) What proportion will continue to burn for at least 1 year?

Solution:

The average value of X is $1/4$, so we let $\lambda = 4$ and $f(x) = 4e^{-4x}$.

- (a) The proportion of light bulbs where X is less than or equal to $1/2$ is

$$P(0 \leq X \leq \frac{1}{2}) = \int_0^{1/2} 4e^{-4x} dx = -e^{-4x} \Big|_0^{1/2} = -e^{-2} + 1 = 0.86466$$

- (b) The proportion of light bulbs that do not burn out for at least 1 year is

$$P(1 \leq X < \infty) = \int_1^{\infty} 4e^{-4x} dx = \lim_{b \rightarrow \infty} \int_1^b 4e^{-4x} dx = \lim_{b \rightarrow \infty} (-e^{-4b} + e^{-4}) = e^{-4} = 0.01832$$

Your Assignment

Student will choose a type of bulb (incandescent, fluorescent, halogen, etc.) and finds out the typical average rated life for that type of bulb. Then through an integration process, the student will

- Find the proportion of the lightbulbs that will burn out within $1/4$ year.
- Find what proportion will continue to burn for at least 2 years.

