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Investigating Series

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Investigating Series

A Mini Project for Module 3

Project Description

This project demonstrates the following concepts in integral calculus:

1. Sequences.
2. Series.
3. Taylor polynomials.
4. Convergence of sequences.

Project description

This project emphasizes pattern recognition and exploration with sequences and series. It also has students work with multiple representations of sequences, namely, graphical and numerical representations. In this project, you will experiment with some infinite sequences and their limits.

Part 1

Starting with a given sequence of numbers, {b1, b2, ...}, you will construct a new sequence $\{a_1, a_2, ...\}$ as follows:

$a_1 = b_1$

$a_2 = b_2 - b_1$

$a_3 = b_3 - b_2$

.
.
.

$a_n = b_n - b_{n-1}$

.
.
.

Starting with each of the following sequences as $\{b_n\}$:

(1) 2/1, 8/3, 26/9, 80/27, 242/81, 728/243, 2186/729, ... and
(2) ¾ , 6/6, 9/8, 12/10, 15/12, 18/14, 21/16, ...

1. Compute the first 6 elements of the sequences $\{a_n\}$.
2. Graph a_n versus n and b_n versus n on the same coordinate axes. Plot at least the first 6 values for each sequence. Visually determine the limit of each sequence, if it exists, and place it on the same graph as a horizontal asymptote.
3. Find an expression for a_n and for b_n in terms of n.
4. Compute the limit of $\{a_n\}$ and $\{b_n\}$ as n->∞. Compare with the limits you found in 2.
5. The definition above gives a_n in terms of b_n and b_{n-1}. Using this definition, write b_n in terms of just the a_i's.
6. Using your answer to 5 to explain in your own words how the sequence $\{a_n\}$ is related to the sequence $\{b_n\}$.
7. Explain in your own words how the limit of $\{b_n\}$ as n->∞ is related to the sequence $\{a_n\}$.

Part 2

Now let $\{a_n\}$ be the sequence $\{\frac{1}{n+1}\}$. Define a new sequence $\{s_n\}$ by

$s_n = a_1+a_2+a_3+...+a_n$.

1. For each of $n = 1$ to $n = 8$, compute the value of s_n. Now look at s_1, s_2, s_4, s_8, and the values of s_n in the table below. How does the value of s_n change when n is doubled?

n	s_n
1	0.5
2	0.833333
4	1.283333
6	1.828968
16	2.439553
32	3.088798
64	3.759276
128	4.440899
256	5.128236

512	5.818466
1024	6.510151

2. Draw the graphs of ln(x+1) and s_n on the same set of axes. What do you see? Can you explain it?
3. On a second set of axes, draw a picture of the function $y = \frac{1}{x+1}$. Find a way to represent the value of ln(n+1)-ln2 in this picture for n=8.
4. Again, on the same (second) graph, find a way to represent the value of s_n. Is there any relationship between s_n and ln(n+1)-ln2? If so what is the relationship?
5. What is the limit of { s_n } as n->∞? Why do you think so?