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The Consumers' and Producers' Surplus

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The Consumers' and Producers' Surplus

A Mini Project for Module 2

Project Description

This project demonstrates the following concepts in integral calculus:

1. Riemann sums.
2. Data modeling
3. Area between curves

Project description.

In a competitive market, the relationship between supply and demand is one of the factors that determine prices of products. Let the following equations describe the relationship between price p and demand $D(x)$, and the relationship between price p and supply S(x) for the supply of x units,

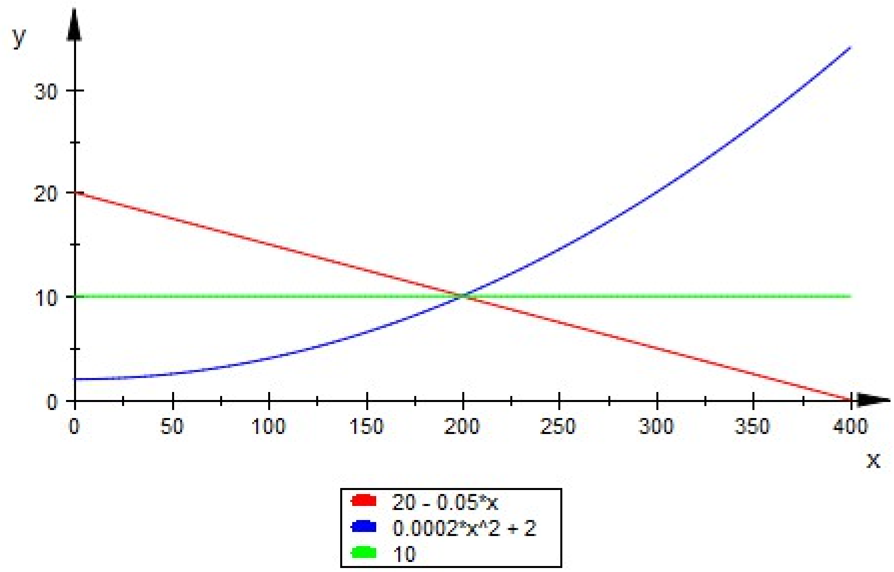
$$p = D(x)$$
$$p = S(x)$$

If $(\bar{x}, \bar{p})$ is the point of intersection of the graphs of these two equations, then $\bar{x}$ is called the equilibrium quantity and $\bar{p}$ is called the equilibrium price. This means that there is no incentive to changing the number of supplied units will either increase the demand but decrease the supply or decrease the demand and increase the supply, thus create unstable market.

Assume

$$p = D(x) = 20 - 0.05x$$
$$p = S(x) = 2 + 0.0002x^2$$

Then by equating $D(x) = S(x)$ and solving for x, the equilibrium quantity $\bar{x} = 200$ ($\bar{x} = -450$ is an extreneous solution), and the equilibrium price $\bar{p} = 10$. Here is a Mupad plot demonstrating the equilibrium price.



Once the equilibrium price $\bar{p}$ is established, it determines consumers' surplus and producers' surplus.

Consumers' surplus *CS* is the total savings to consumers who are willing to pay more than the equilibrium price $\bar{p}$, but are still able to buy the product for $\bar{p}$.

$$CS = \int_0^{\bar{x}} (D(x) - \bar{p})dx.$$

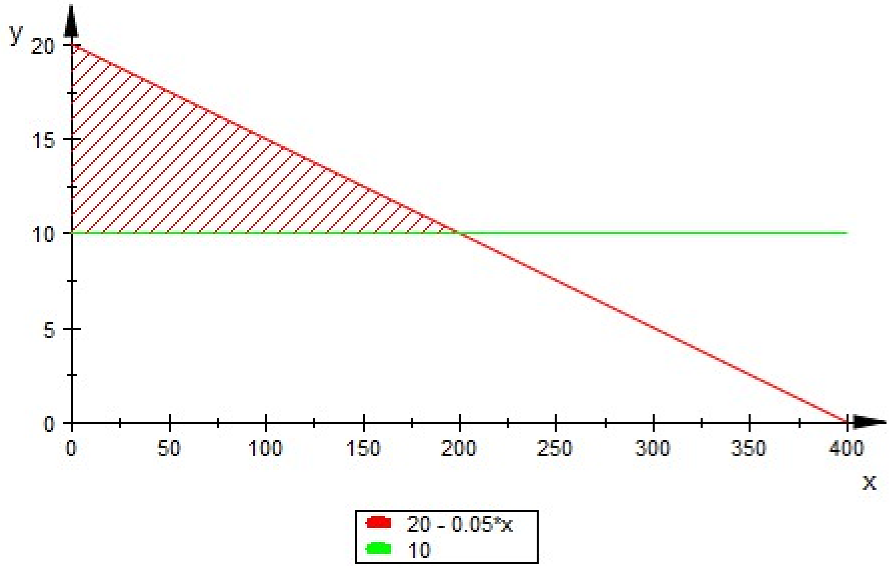
To justify this formula consider the supplies are between x_k and $x_k + \Delta x$. Then a customer who can pay the demand price $D(x)$ for x between x_k and $x_k + \Delta x$ save approximately $D(x_k) - \bar{p}$ per unit supplied. Savings per Δx unit supplied are

$$(approximate\ savings\ per\ unit) \times (number\ of\ units) = (D(x_k) - \bar{p})\Delta x.$$

Adding up over a partition $x_1, x_2 = x_1 + \Delta x, x_3 = x_2 + \Delta x,\ x_n = x_{n-1} + \Delta x$, we obtain a Riemann sum converging to the indicated integral

$$\sum_{k=1}^{n}(D(x_k)-\bar{p})\Delta x \to \int_0^{\bar{x}}(D(x)-\bar{p})dx$$

In our example



$$\begin{aligned} CS &= \int_0^{200}(20-0.05x-10)dx \\ &= \int_0^{200}(10-0.05x)dx \\ &= [10x-0.025x^2]_0^{200} \\ &= 2{,}000-1{,}000=1{,}000, \end{aligned}$$

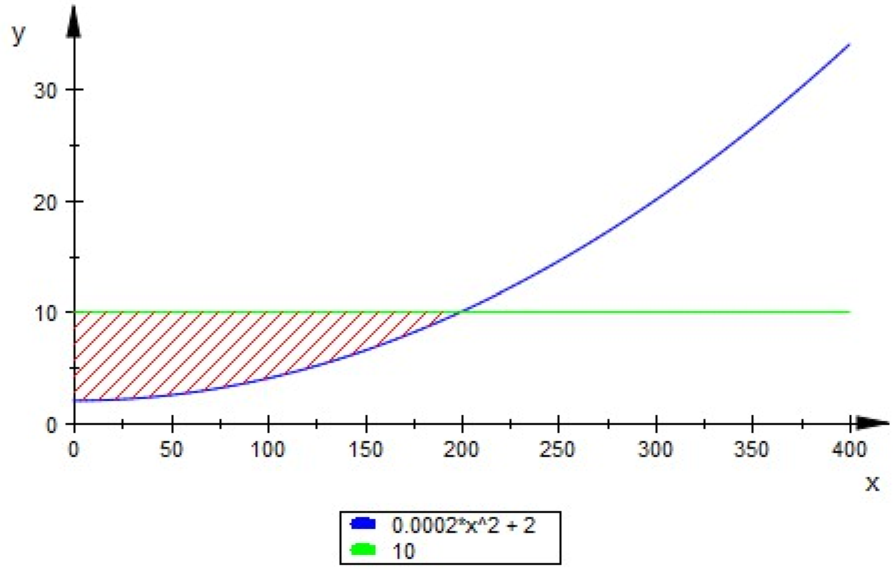
would be saved by any customer willing to pay more than the equilibrium price of \$10 but still being able to buy the product for \$10.

Producers' surplus PS is the total gain to producers who are willing to supply units at a lower price than the equilibrium price $\bar{p}$, but are still able to supply units at $\bar{p}$.

$$PS = \int_0^{\bar{x}}\big(\bar{p}-P(x)\big)dx.$$

Your assignment. Justify the integral formula for PS using Riemann sums.

In our example



$$PS = \int_0^{200} \left(10 - (2 + 0.0002x^2)\right)dx$$
$$= \int_0^{200} (8 - 0.0002x^2)dx$$
$$= \left[8x - 0.0002\frac{x^3}{3}\right]_0^{200}$$
$$= 1{,}600 - \frac{1{,}600}{3} \approx 1{,}067,$$

would be gained by any producer willing to supply for less than the equilibrium price of $10 but still being able to supply the product for $10.

Mupad Code for graphs.

```
Dem:= x -> 20 - 0.05*x:
Sup:= x -> 2+0.0002*x^2:
pbar := 10:
f1:= plot::Function2d(Dem(x), x = 0 .. 400, Color = RGB::Red,
        LegendText = expr2text(Dem(x))):
f2:= plot::Function2d(Sup(x), x = 0 .. 400, Color = RGB::Blue,
        LegendText = expr2text(Sup(x))):
f3:= plot::Function2d(pbar(x), x = 0 .. 400, Color = RGB::Green,
        LegendText = expr2text(pbar(x))):
plot(f1,f2,f3, LegendVisible = TRUE):
plot(f1, f3, plot::Hatch(f1, f3, 0 .. 200),LegendVisible = TRUE):
plot(f2, f3, plot::Hatch(f2, f3, 0 .. 200),LegendVisible = TRUE):
```

Your assignment.

1. Assume your own price-demand, and price-supply equations, and find the equilibrium price, and equilibrium quantity. Produce a graph that identifies the consumers' surplus and the producers' surplus. Then calculate the *CS* and *PS* quantities.
2. The following table contains hypothetical price-supply and price-demand data.

x	*p=D(x)*	*p=S(x)*
0	20.1	18.1
5	19.8	18.2
10	19.7	18.4
15	19.1	18.7
20	18.5	19.8

a. Find the equilibrium price, and equilibrium quantity.
b. Produce a graph that identifies the consumers' surplus and the producers' surplus, you can use Excell, for example, or Matlab.
c. Use the left end-point rule to find the consumers' surplus and the producers' surplus.
d. Fit the quadratic regression model for both, the price-demand and price-supply data using for example Excell or Matlab.
e. Find the consumers' surplus and the producers' surplus using Matlab, for example.

3. Find real data for a commodity of your interest and repeat steps (a) – (e) in part **2.**

Agricultural commodities monthly data are available at http://usda.mannlib.cornell.edu/MannUsda/viewDocumentInfo.do?documentID=1194