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Total Cost and Profit

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Total Cost and Profit

A Mini Project for Module 1

Project Description

This project demonstrates the following concepts in integral calculus:

- Indefinite integrals.

Project Description

- Use integration to find total cost functions from information involving marginal cost (that is, the rate of change of cost) for a commodity.
- Use integration to derive profit functions from the marginal revenue functions.
- Optimize profit, given information regarding marginal cost and marginal revenue functions.

The marginal cost for a commodity is $\overline{MC}$ = C′(x), where C(x) is the total cost function. Thus if we have the marginal cost function, we can integrate to find the total cost. That is, C(x) = $\int MC \;\; dx$.

The marginal revenue for a commodity is $\overline{MR}$ = R′(x), where R(x) is the total revenue function.

If, for example, the marginal cost is $\overline{MC}$ = 1.01(x + 190)$^{0.01}$ and

$\overline{MR} = \left(1/\sqrt{2x+1}\right) + 2$, where x is the number of thousands of units and both revenue and cost are in thousands of dollars. Suppose further that fixed costs are $100,236 and that production is limited to at most 180 thousand units.

$$C(x) = \int \overline{MC}\; dx \;= \int 1.01(\mathrm{x}+190)^{0.01}\; dx = (\mathrm{x}+190)^{1.01} + K$$

Now, we know that the total revenue is 0 if no items are produced, but the total cost may not be 0 if nothing is produced. The fixed costs accrue whether goods are produced or not. Thus the value for the constant of integration depends on the fixed costs *FC* of production. We will find the constant of integration K by using the fact that C(0) = FC = 100.236.

$100.236 = C(0) = (190)^{1.01} + K \rightarrow K = =100$

Thus $C(x) = (x+ 190)^{1.01} – 100$

$$R(x) = \int \overline{MR}\ \mathrm{dx} = \int \left[(2\mathrm{x}+1)^{-1/2} + 2\right] dx$$

$$= \frac{\frac{1}{2}(2x+1)^{1/2}}{1/2} + 2x + K$$

R(0) = 0 means

$$0 = (1)^{1/2} + 0 + K, \text{ or } K = -1$$

Thus $R(x) = (2x+1)^{1/2} + 2x -1$

Profit is usually maximized when $\overline{MR} = \overline{MC}$. Please note that this does not always give us a maximum *positive* profit.

If it is difficult to solve $\overline{MR} = \overline{MC}$ analytically, we could use a graphing utility to solve this equation by finding the point of intersection of the graphs of $\overline{MC}$ and $\overline{MR}$. Use the graph to find what level of production yields maximum profit (or minimum loss).

Your Assignment

Your assignment is to find (look up internet) the marginal cost function, the marginal revenue function, and fixed costs of a commodity of your choice. You also have to make a choice for the number of units the production is limited to.

(a) Determine C(x) and R(x).

(b) Graph C(x) and R(x) to determine whether a profit can be made.

(c) Estimate the level of production that yields maximum profit, and find the maximum profit (or minimum loss).